Asymptotically free models with massive vector fields

1975 J. Phys. A: Math. Gen. 8 1814

(http://iopscience.iop.org/0305-4470/8/11/017)

J. Phys. A: Math. Gen., Vol. 8, No. 11, 1975. Printed in Great Britain. © 1975

Asymptotically free models with massive vector fields

E S Fradkin and O K Kalashnikov
I E Tamm Department of Theoretical Physics, P N Lebedev Physical Institute of the USSR Academy of Sciences, Leninskii Prospekt 53, Moscow, USSR

Received 25 April 1975

Abstract. Asymptotically free models with massive vector fields are constructed within the SU(2) group. Physical consequences of such models are briefly discussed.

1. Introduction

A number of renormalizable models of universal interaction (see eg reviews by Fradkin and Tyutin 1974, Weinberg 1974) using Yang–Mills fields as intermediate ones have been built in recent years. In such theories initial massless fields become massive due to the interaction with scalar fields by means of the Higgs mechanism. It is well known however that the theory of non-abelian gauge fields is not only renormalizable but is also referred to the class of so called asymptotically free theories (Gross and Wilczek 1973a, b, Politzer 1973, Fradkin and Kalashnikov 1974). The 'zero charge' difficulty (Landau and Pomeranchuk 1955, Fradkin 1955) is absent from these theories and in this sense they are mathematically consistent. In this connection one could believe that the interaction of Yang–Mills fields with other fields within universal models offers the possibility in principle of achieving asymptotic freedom for the theories with massive fields on the whole. The models thus built would represent a consistent theory of quantum fields in which both the 'zero charge' difficulty and infrared divergences are absent. This question was recently analysed by Cheng *et al* (1974) where the authors however came to the conclusion that all the necessary fields cannot become massive if one bears in mind the preservation of asymptotic freedom for the model on the whole.

Below it will be shown however that generally speaking it is not quite so. In fact a number of models could be pointed out where the requirement of asymptotic freedom for the whole model does not prevent all the necessary particles from acquiring masses. Such models can be built with a definite choice of multiplets of interacting fields and coupling constants only. The definite relations between coupling constants open the possibility of finding here solutions of the renormalization group equations which correspond to ultraviolet-unstable fixed points. With these solutions we find that the models under consideration are asymptotically free. The importance of such solutions has already been mentioned by Chang (1974) and Suzuki (1974). In the article by Cheng *et al* (1974) such solutions are not discussed. Here we shall restrict ourselves to consideration of the simplest models of this kind within the SU(2) group only.

2. Asymptotically free gauge models

2.1. A first model

Let us first consider the theory with a massless gauge field W_μ^a whose Lagrangian, in the usual notation, has the following form:

$$\begin{aligned}\mathcal{L} = & -\tfrac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_0\epsilon^{abc}W_\mu^b W_\nu^c)^2 - \tfrac{1}{2}[(\partial_\mu\delta^{ac} + g_0\epsilon^{abc}W_\mu^b)\phi^c]^2 - \mu^2\phi^a\phi^a \\ & + \sum_{\sigma=1}^{2}\{\overline{\Psi}_\sigma^a[\gamma_\mu(\partial_\mu\delta^{ac} + g_0\epsilon^{abc}W_\mu^b) + M_\sigma\delta^{ac}]\Psi_\sigma^c\} + \bar{S}_\sigma(\gamma_\mu\partial_\mu + M_\sigma)S_\sigma - \mathrm{i}\eta_0\epsilon^{abc} \\ & \times \sum_{\sigma=1}^{2}\overline{\Psi}_\sigma^a\phi^b\Psi_\sigma^c - h_0\sum_{\sigma=1}^{2}[(\overline{\Psi}_\sigma\phi)\tfrac{1}{2}(1+\gamma_5)S_\sigma + \tfrac{1}{2}\bar{S}_\sigma(1-\gamma_5)(\phi\Psi_6)] \\ & - \tfrac{1}{8}\lambda_0^2(\phi^a\phi^a)^2.\end{aligned}$$

Here $g_{\mu\mu} = (-+++)$, γ matrices are chosen in the Schwinger representation $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$, and S is a singlet field. With the appropriate choice of multiplets, masses and coupling constants this model may be identified with a lepton part of the Georgi and Glashow (1972) (GG) unified model of weak and electromagnetic interactions, although some other interpretations of model (1) are also possible. The fact should be emphasized that among unified models of weak and electromagnetic interactions within the SU(2) group the GG model is the only one possessing an asymptotic freedom. The alternative unified models of weak and electromagnetic interactions of the Weinberg–Salam type, which are often used as a part of universal models for groups of higher symmetry, are not asymptotically free due to the presence of an abelian gauge field of group U(1).

We would like to discuss here in more detail than has been done by Chang (1974), what physical consequences for the GG model would result from the requirement of asymptotic freedom. We shall destroy, in particular, the μ–e invariance of the initial model and have new mass relations by requiring that light lepton masses correspond to electron and muon real masses. Besides we shall present here an additional solution of the renormalization group equations which is interesting in itself.

Equations of the renormalization group for this model have the following form:

$$\begin{aligned} 16\pi^2\,\mathrm{d}g^2/\mathrm{d}t &= -\tfrac{10}{3}g^4 \\ 16\pi^2\,\mathrm{d}\eta^2/\mathrm{d}t &= 24\eta^4 + 5h^2\eta^2 - 24g^2\eta^2 \\ 16\pi^2\,\mathrm{d}h^2/\mathrm{d}t &= 16h^4 + 10h^2\eta^2 - 12g^2h^2 \\ 16\pi^2\,\mathrm{d}\lambda^2/\mathrm{d}t &= 11\lambda^4 + (32\eta^4 + 16h^2 - 24g^2)\lambda^2 + 24g^4 - 64\eta^4 - 32h^4. \end{aligned} \tag{2}$$

The solution of this set of equations corresponding to ultraviolet-stable fixed points cannot be found in the class of solutions characteristic for asymptotically free theories. This was just the result pointed out by Cheng *et al* (1974). However, if quite definite relations exist between coupling constants, the set of equations (2) turns out to have solutions with which the considered model is asymptotically free. In this case fixed points (all of them or at least one) are ultraviolet-unstable, but it can be shown (Suzuki 1974) that it is not an obstacle for the formulation of a consistent asymptotically free theory. Here we have found two different sets of relations between the coupling constants

$$\eta^2 = (\bar{\eta}g)^2 = \tfrac{431}{3\cdot 167}g^2 \qquad h^2 = (\hbar g)^2 = \tfrac{2}{3\cdot 167}g^2$$
$$\eta^2 = (\bar{\eta}g)^2 = \tfrac{31}{36}g^2 \qquad h^2 = 0, \tag{3}$$

with which the renormalization group equations have two independent solutions corresponding to the asymptotic freedom of the considered model. The first of them will be used below in the discussion of the physical consequences of the requirement of asymptotic freedom in the GG model. The other solution makes it possible to formulate an asymptotically free quantum electrodynamics. The latter has the same experimental predictions as ordinary quantum electrodynamics up to the energy corresponding to the mass of intermediate heavy particles but the 'zero charge' difficulty is absent here.

Thus after carrying out the usual renormalization programme and fulfilment of relations (3) between renormalized coupling constants the model (1) will be asymptotically free. At the same time the presence in this model of a scalar field with self-action makes it possible to supply all the necessary fields with masses by means of the Higgs mechanism.

In particular, if we identify the model (1) with the lepton part of the GG unified model of weak and electromagnetic interactions, the masses of the corresponding fields will have the following forms:

$$m_W = 53{\cdot}0\sqrt{(\sin\beta_e \sin\beta_\mu)}$$
$$m_{E^+} = m_W(\hbar\cot\beta_e + \bar{\eta}) \qquad m_{M^+} = m_W(\hbar\cot\beta_\mu + \bar{\eta}) \tag{4}$$
$$m_{E^0} = \hbar m_W/\sin\beta_e \qquad m_{M^0} = \hbar m_W/\sin\beta_\mu .$$

Here E^+, M^+ are heavy leptons; E^0, M^0 are massive neutral particles; β_e, β_μ are mixing angles of the electron and muon neutrino with the corresponding neutral particle; $\hbar$, $\bar{\eta}$ are as defined in formula (3). The electron and muon neutrino masses are equal to zero. This latter requirement leads here to the connection between bare masses of fermion multiplets and $\langle\phi\rangle$ which results in the appearance in the theory of a number of mass relations:

$$m_{E^+} + m_{e^-} = 2m_{E^0}\cos\beta_e \qquad m_{M^+} + m_{\mu^-} = 2m_{M^0}\cos\beta_\mu . \tag{5}$$

We destroy here the μ–e invariance of the model and identify light lepton masses with electron and μ meson masses:

$$m_{e^-} = m_W(\hbar\cot\beta_e - \bar{\eta}) \qquad m_\mu = m_W(\hbar\cot\beta_\mu - \bar{\eta}). \tag{6}$$

The latter uniquely defines the angles β_e and β_μ. Thus as distinct from the usual GG model formulation the masses of the W boson and other particles are no longer arbitrary and have quite definite values:

$$m_W = 3{\cdot}54\ \mathrm{GeV}/c^2 \qquad m_{E^+} = 6{\cdot}58\ \mathrm{GeV}/c^2 \qquad m_{E^0} = 3{\cdot}30\ \mathrm{GeV}/c^2$$
$$m_{M^+} = 6{\cdot}68\ \mathrm{GeV}/c^2 \qquad m_{M^0} = 3{\cdot}40\ \mathrm{GeV}/c^2. \tag{7}$$

This interesting fact is a direct consequence of the requirement of asymptotic freedom within the GG model which finds its expression in the condition (3) and mass relations (4, 5). From the physical point of view this result should rather be considered as unsatisfactory though the likeness of masses of recently discovered ψ particles to those of the W boson and Higgs boson in the theory under consideration is tempting.

2.2. *A second model*

The second model discussed hereafter is built in a similar way:

$$\begin{aligned}\mathscr{L} = & -\tfrac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_0\epsilon^{abc}W_\mu^b W_\nu^c)^2 - |(\partial_\mu\delta^{ac} - \mathrm{i}g_0\tfrac{1}{2}\tau_{ac}^b W_\mu^b)\phi^c|^2 - \mu^2\phi^{+,a}\phi^a \\ & + \bar{\Sigma}^a[\gamma_\mu(\partial_\mu\delta^{ac} + g_0\epsilon^{abc}W_\mu^b) + M_\Sigma\delta^{ac}]\Sigma^c + \sum_{\sigma=1}^{2} \bar{\psi}_\sigma^a[\gamma_\mu(\partial_\mu\delta^{ac} - \mathrm{i}\tfrac{1}{2}\tau_{ac}^b W_\mu^b) \\ & + M_\sigma\delta^{ac}]\psi_\sigma^c - \kappa_0[\bar{\Sigma}^d(\phi^{+,a}\tfrac{1}{2}\tau_{ab}^d N^b) + (\bar{N}^a\tfrac{1}{2}\tau_{ab}^d\phi^b)\Sigma^d] \\ & - \kappa_0[\bar{\Sigma}^d(\phi_c^{+,a}\tfrac{1}{2}\tau_{ab}^d\Xi^b) + (\bar{\Xi}^a\tfrac{1}{2}\tau_{ab}^d\phi_c^b)\Sigma^d] - \tfrac{1}{2}\lambda_0^2(\phi^{+,a}\phi^a)\end{aligned} \tag{8}$$

but as distinct from the first model the SU(2) group is completely destroyed here. The latter is achieved due to introduction into the theory of a complex scalar field ϕ^a which interacts with the gauge field and fermion multiplets of real particles. Here $\psi_1 \equiv N$ is a nucleon doublet; ψ_2 is a doublet of Ξ particles; and Σ is a triplet of Σ particles which can also contain a Λ_0 singlet in combination with Σ_0 ($\Sigma_0 \sin\beta + \Lambda_0\cos\beta$). $\phi_c = C\phi^+$ where $\mathrm{i}C = \tau_2$ is the charge symmetry matrix. After broken symmetry there are no massless particles and mass relations will be so chosen that this model could be interpreted as the simplest rough model imitating a strong hadron interaction. The inclusion of a π meson into this model does not produce difficulties but because it is cumbersome it will be published in a separate paper.

The renormalization group equations for this model have a rather simple form:

$$\begin{aligned}16\pi^2\,\mathrm{d}g^2/\mathrm{d}t &= -\tfrac{19}{3}g^4 \\ 16\pi^2\,\mathrm{d}\kappa^2/\mathrm{d}t &= \tfrac{33}{2}\kappa^4 - \tfrac{27}{4}\kappa^2 g^2 \\ 16\pi^2\,\mathrm{d}\lambda^2/\mathrm{d}t &= 12\lambda^4 + (12\kappa^2 - 9g^2)\lambda^2 + \tfrac{9}{4}g^4 - 6\kappa^4\end{aligned} \tag{9}$$

and their solution is easily found. The solution obtained corresponds to the ultraviolet-unstable fixed points of the set of equations (9) and is realized if the following simple relation:

$$\kappa^2 = (\bar{\kappa}g)^2 = \tfrac{122}{81}g^2 \tag{10}$$

holds for the coupling constants. The real model possesses an asymptotic freedom and all the fields in it are massive.

Vector field mass m_W results from spontaneous symmetry breaking

$$\langle\phi^a\rangle = \langle\phi^{+,a}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \xi\end{pmatrix} \tag{11}$$

and is the same for all components of the vector multiplet. After an appropriate choice of ξ we may identify $m_\rho = \frac{1}{2}g\xi/\sqrt{2}$ with the mass of ρ mesons. Hadron mass is obtained by diagonalization of the mass matrix and in a general case of different bare masses of hadron multiplets it is defined by the following equations:

$$\begin{aligned}m_{\mathrm{p},\Sigma^+} &= \tfrac{1}{2}(M_\mathrm{N} + M_\Sigma) \pm \{[\tfrac{1}{2}(M_\Sigma - M_\mathrm{N})]^2 + \Delta^2\}^{1/2} \\ m_{\Xi^-,\Sigma^-} &= \tfrac{1}{2}(M_\Xi + M_\Sigma) \pm \{[\tfrac{1}{2}(M_\Xi - M_\Sigma)]^2 + \Delta^2\}^{1/2} \\ (M_\mathrm{N} - \lambda)(M_\Sigma - \lambda)&(M_\Xi - \lambda) - \Delta^2[\tfrac{1}{2}(M_\Xi + M_\mathrm{N}) - \lambda] = 0\end{aligned} \tag{12}$$

where $\Delta = \frac{1}{2}\kappa\xi = \bar{\kappa}m_\rho\sqrt{2}$ and $\bar{\kappa}$ is determined by equation (10). In a particular case, when $\frac{1}{2}(M_\Xi + M_\mathrm{N}) = M_\Sigma$, equation (12) is solved in an explicit form and the hadron mass spectrum takes the following form:

$$\begin{bmatrix} m_{\mathrm{p}} \\ m_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(M_\Sigma + M_\mathrm{N}) - \{[\frac{1}{2}(M_\Sigma - M_\mathrm{N})]^2 + \Delta^2\}^{1/2} \\ M_\Sigma - \{[\frac{1}{2}(M_\Xi - M_\mathrm{N})]^2 + \Delta^2\}^{1/2} \end{bmatrix}$$

$$\begin{bmatrix} m_{\Sigma^+} \\ m_{\Sigma^0} \\ m_{\Sigma^-} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(M_\Sigma + M_\mathrm{N}) + \{[\frac{1}{2}(M_\Sigma - M_\mathrm{N})]^2 + \Delta^2\}^{1/2} \\ M_\Sigma \\ \frac{1}{2}(M_\Xi + M_\Sigma) - \{[\frac{1}{2}(M_\Xi - M_\Sigma)]^2 + \Delta^2\}^{1/2} \end{bmatrix}$$

$$\begin{bmatrix} m_{\Xi^0} \\ m_{\Xi^-} \end{bmatrix} = \begin{bmatrix} M_\Sigma + \{[\frac{1}{2}(M_\Xi - M_\mathrm{N})]^2 + \Delta^2\}^{1/2} \\ \frac{1}{2}(M_\Xi + M_\Sigma) + \{[\frac{1}{2}(M_\Xi - M_\Sigma)]^2 + \Delta^2\}^{1/2} \end{bmatrix}.$$

There is no sense in calculating the numerical values of masses, since after π mesons are introduced into the model, Δ diminishes considerably due to the decrease of $\bar{\kappa}$. Nevertheless it is seen that with an appropriate choice of Δ and bare masses the masses of the corresponding multiplets can be quite satisfactory.

3. Conclusion

On the whole the results obtained in the present paper can be formulated as follows: the requirement of asymptotic freedom imposes considerable limitations on the class of possible models; nevertheless even within the SU(2) group it is quite possible to build a number of asymptotically free realistic models in which all the necessary particles are massive. For the groups of higher symmetry realization of such models will be still more satisfactory from a physical point of view.

Acknowledgments

The authors would like to thank Dr B L Voronov and Dr I V Tyutin who express the same ideas.

References

Chang N-P 1974 *Phys. Rev.* D **10** 2706
Cheng T P, Eichten E and Li L-P 1974 *Phys. Rev.* D **9** 2259
Fradkin E S 1955 *Zh. Eksp. Teor. Fiz.* **28** 750
Fradkin E S and Kalashnikov O K 1974 *Lett. Nuovo Cim.* **10** 771
Fradkin E S and Tyutin I V 1974 *Riv. Nuovo Cim.* **4** 1
Georgi H and Glashow S L 1972 *Phys. Rev. Lett.* **28** 1494
Gross D J and Wilczek F 1973a *Phys. Rev. Lett.* **30** 1343
—— 1973b *Phys. Rev.* D **8** 3633
Landau L D and Pomeranchuk I Y 1955 *Dokl. Akad. Nauk.* **102** 489
Politzer H D 1973 *Phys. Rev. Lett.* **30** 1346
Suzuki M 1974 *Nucl. Phys.* B **83** 269
Weinberg S 1974 *Rev. Mod. Phys.* **46** 255